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OPTIMIZATION METHODS IN MODELING THE MECHANICAL PROPERTIES OF HEAVY STEEL PLATES

METODY OPTYMALIZACYJNE W MODELOWANIU WŁASNOŚCI MECHANICZNYCH BLACH GRUBYCH

The paper is devoted to an optimization approach to a problem of statistical modeling of mechanical properties of heavy steel plates during a real industrial manufacturing process. The approach enables the manufacturer to attain a specific set of the final product properties by optimizing the alloying composition within the grade specifications. Because this composition has to stay in the agreement with earlier indicated specifications, it leads to the large system of linear constraints, and the problem itself can be expressed in the form of linear programming (LP) task. It turns out however, that certain of the constraints contain the coefficients which have to be estimated on the base of the data gathered in the production process and as such they are uncertain. Consequently, the initial optimization task should be modeled as so-called Chance Constrained Programming problem (CCP), which is a special class within the stochastic programming problems. The paper presents mathematical models of the optimization problem that result from both approaches and indicates differences which are important for the decision makers in the production practice. Some examples illustrating the differences in solutions resulting from LP and CCP models are presented as well. Although the statistical analysis presented in this paper is based on the data gathered in the ISD Czestochowa Steelworks, the proposed approach can be adopted in any other process of steel production.
Keywords: heavy steel plate, mechanical properties, optimization, chance constrained programming

Praca poświęcona jest optymalizacyjnemu podejściu do modelowania wybranych własności mechanicznych blach grubych w trakcie rzeczywistego procesu produkcyjnego. Proponowane podejście umożliwia producentowi uzyskanie ustalonego zbioru własności blach stalowych z jednoczesną optymalizacją ich składu chemicznego w zakresie określonym poprzez normy definiujące dany gatunek. Ponieważ ów skład musi pozostawać w zgodzie z wcześniej wskazanymi w zamówieniu specyfikacjami prowadzi to do dużego układu liniowych ograniczeń, a sam problem można wyrazić w postaci zadania programowani liniowego. W praktyce okazuje się, że pewne z tych ograniczeń zawierają współczynniki, które muszą być oszacowane na podstawie danych zebranych w procesie produkcyjnym i jako takie ich wartości są niepewne. W konsekwencji wyjściowy problem powinien zostać sformułowany jako zadanie programowania stochastycznego.

W pracy przedstawia się modele optymalizacyjne wynikające z obu podejść i wskazuje na różnice ważne z punktu widzenia podejmowania decyzji w praktyce produkcyjnej. Prezentowane są również przykłady ilustrujące różnice w rozwiązaniach uzyskiwanych z pomocą tych dwóch modeli. Chociaż analiza statystyczna przedstawiona w pracy oparta jest na danych zebranych w ISD Huta Częstochowa, to proponowane podejście może być zaadaptowane w dowolnym innym procesie produkcji stali.

1. Introduction

Contemporary global steel industry is facing significant pressure to achieve operational excellence and improve profitability. Any competitive metallurgical company should have the ability to allow customers to design their own alloy and have it melted at cost-effective prices. Thus manufacturers of the heavy steel plates must possess proper tools in order to track and control the production, modify the technology and forecast various technological features of the final product. Creation of mathematical models for phenomena occurring during the technological process is the important stage of the realization of this goal. These models contribute to the decrease of general production costs and increase of the product competitiveness thanks to the control of production process and improvement of the final product quality. It turns out in practice that in order to match the requirements of the contemporary market such models, beside the supervisory tasks should also fulfill optimizing functions. Papers devoted to optimization aspects of various problems related to the metallurgical industry appear in the literature more and more often, see e.g. [1-7], and this paper is also devoted to these class of

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problems. The optimization models which we formulate here can be used in the stage of modeling the mechanical properties of heavy steel plates during the real production process. The proposed approach enables the optimization of chemical composition of the steel alloy and some technological parameters with the simultaneous fulfillment of earlier imposed requirements for final properties of the product. These requirements usually result from the steel grade standards but can also be connected with certain additional requirements imposed by the customer. All these predefined requirements yield a set of constraints which should be satisfied by the acceptable solutions. It turns out that usually these limitations can be expressed through linear functions, and the problem itself can be formulated in the form of Linear Programming (LP) model. However, it appears that certain constraints should be expressed by relations, which true quantitative nature remains unknown. Especially it refers to the constraints connected with the final mechanical properties of the product. To state them properly we need good mathematical models describing the impact of the alloy composition and rolling process parameters on the final properties of the steel plates. The metallurgical processes, however, are so complex that it is often impossible to describe them by the formal-analytic expressions. An important and helpful alternative approach in such cases is the statistical analysis of the industrial data. In the context of the analysis of metallurgical processes we can find various methods of data exploration described in the literature, [8-10]. One of the most popular data-mining methods is the multidimensional regression analysis. It was applied successfully in the examination of many relations in the metallurgical industry, [11-14]. In the considered case an additional advantage of this approach is that it leads to linear constraints and our initial problem stays in the class of LP problems. However, due to the nature of any statistical analysis, the coefficients in the constraints resulting from the regression analysis are known only approximately. This should be reflected in the mathematical model of the optimization problem. Therefore the final optimization task is formulated as the stochastic programming problem, more precisely, in the form of so-called Chance Constrained Program, [15-17].

The statistical analysis presented in this paper is based on the data gathered during the real manufacturing process in the ISD Czestochowa Steelworks. However, the proposed optimization approach to modeling the final mechanical properties of the steel plates can be adopted in any other steel production process.

2. General description of the optimization problem

It was emphasized in the introduction, that for contemporary steel industry it is necessary to produce new generations of steel products with the help of high performance low cost economical technologies. To meet the steel market requirements the manufacturers need an optimization models, that take into account many different constraints resulting from the technological standards, customers expectations and limitations of the implemented technology. Such models would enable the manufacturer to attain a specified set of final properties by optimizing the alloying within grade composition and other predefined requirements. Generally the constraints that have to be fulfilled are of the following types: chemical composition constrains (CC), constraints connected with the parameters defining the realization of the technological process (TP), and constraints connected with the steel plate final properties (SPFP).

The CC constrains can be expressed in the form : $CE_i \geqslant a_i$ and/or $CE_i \leqslant b_i$, $i = 1,...n$, where CE_i stands for the concentration of i–th chemical element or a linear combination of such elements. An example of the latter is the well-known Carbon Equivalent Value (CEV) which appears in many steel grades definitions.

Parameters defining the realization of the technological process are primarily connected with the rolling mill construction and the implemented automatic control system. They yield TP constraints, which result from the feasibilities and limitations of the system. All the TP constraints can be expressed in the linear form, because all of them are defined by given limit values which, in order to achieve a desired final result, should not be crossed by particular technological parameters.

From the market standpoint perhaps the most important are the final properties of the product and thus the SPFP constraints demand the particular attention. The analysis presented in this paper is primarily concerned with the mechanical properties of the steel plates described by the following three parameters: yield strength (*Reh*), tensile strength (*Rm*), elongation (*A*). The constraints connected with these parameters result from the steel grade characteristics and, possibly, from the customer specifications. However the main problem is that these parameters cannot be observed directly during the production process and thus any constraints connected with them cannot be stated explicitly in the optimization model. Thus, first we should develop models relating the final mechanical properties of the steel plate to the chemical composition of the steel as well as the production process parameters. As we pointed out in the introduction, an important method of obtaining satisfactory models in terms of production control and forecasting is a multidimensional regression analysis. In the context of predicting mechanical properties of steel products it was used e.g. in [4,12,13,14,18]. In the next section we develop regression models which can be used for the SPFP constraints statement.

In every mathematical model of real optimization problem the basic issue is the establishment of the quality criterion for the solutions, i.e. the definition of the objective function. In our paper, similarly as e.g. in [1,6], it will be the cost criterion. However, in order to make the obtained models possibly general, we define the cost quite widely. The cost function (CF) is expressed by a linear function of various alloy composition elements, which are multiplied by appropriate weights representing their relative impact on the total cost. i.e. it takes the form

$$CF = \sum_{i=1}^{m} w_i CE_i \tag{1}$$

where CE_i, as previously, stands for the concentration of the $i-$th chemical element while w_i is its relative weight. Such objective function can encompass both, directly the price of admixtures as well as e.g. the cost of technological process leading to achieving a desired level of a given element concentration. The most important components determining the total cost as well as the adequate weights are subject of the manufacturer experts choice and in practice they should be assigned by a production manager.

3. Regression models in SPFP constraints formulation

Regression analysis allow to describe the relationship between the input and output variables, without going into the essence of the phenomena occurring during the whole process. In this part of the paper we make use of the regression methodology in order to formulate the SPFP constraints. Thus we need to develop regression models describing the relation between the final mechanical properties of the steel plates and the variables which can be tracked and controlled during the production process. A general form for the functional relations we are looking for can be given as follows:

$$par = \beta_0 + \sum_{i=1}^{k} \beta_i X_i + Z \tag{2}$$

In the above formula the parameter *par* represents the yield *Reh*, the tensile strength *Rm*, or the elongation *A*, dependently on the model, the regressors X_i's are the technological characteristics of the rolling process and the concentration of chemical elements reflecting the chemical composition of the considered steel grade, Z stands for the random disturbance of a given model i.e. it represents the impact of all factors not included explicitly in the model.

3.1. Description of manufacturing process and the data

The regression models presented below are built on the base of the data gathered during the industrial manufacturing process. Heavy plate production line in ISD Czestochowa Steelworks consists of the following sections: steel castings, preparation of furnace charge, preparation in heating furnaces, rolling, steel plates finishing, corrosion protection and the quality control. The whole production line is managed by various computer systems. The most important ones are the following: automation system for steel continuous casting, steelworks management system, system for tracking and controlling the rolling process, general production tracking and quality control system in the rolling department. Data collected in these systems describe the actual manufacturing process, starting from the steelworks and ending up on final product.

Because the final form of regression models significantly depend on *the method of the rolling technology*, we present here models which were developed on the base of the data collected during the production process adopting a rolling scheme called *rolling with controlled finishing rolling temperature*, referred to as CFRT method. It is one of the most frequently used rolling schemes in the ISD Czestochowa Steelwork manufacturing practice.

The statistical analysis described in this section is based on a data set containing about 8 000 records drawn from a whole database collected between 2008 and 2010. The set was split in half. The first part was used for *models development*, while the second one formed a base for the *prediction error analysis*. The sets will be referred to as MD and PEA sets, respectively. Each record in the data sets contains 38 values of various characteristics of the production process and the steel alloy. These variables may be considered as potential regressors in our models. As it was pointed out in the literature, see e.g. [8,19-21], the industrial data are typically noisy and can be highly collinear. Thus the data was carefully examined especially for the elimination of weak data points such as outliers and leverage (influential) points, see [19,22]. In this study an observation was considered as an outlier if the absolute value of the related residual was at least three times greater than the residuals sample standard deviation. It turns out, that this condition was satisfied only in 65 cases.

Some fundamental statistical characteristics of the quantities which are finally used as regressors in the developed models are presented in Table 1.

TABLE 1
Statistical characteristics of the empirical data – CERT method of rolling

Explanatory variable	Min	Max	Mean	Standard Deviation
Th [mm]	6.0	111.0	26.93	16.86
FRT [°C]	755	990	852.2	28.9
C [%]	0.09	0.31	0.151	0.021
Mn [%]	0.45	1.60	1.22	0.31
Si [%]	0.14	0.49	0.272	0.057
Nb [%]	0.001	0.040	0.006	0.008
Cr [%]	0.02	0.52	0.048	0.047
Ni [%]	0.048	0.265	0.073	0.014
Ti [%]	0.001	0.015	0.0029	0.0011
P [%]	0.007	0.024	0.013	0.0026
V [%]	0.001	0.119	0.004	0.005

The denotation in Table 1 is the following: *Th* – final thickness, *FRT* – finishing rolling temperature, and all the remaining symbols denote chemical elements.

3.2. Regression models

To determine the final set of explanatory variables we use the so-called Hendry's approach, also known as *general-to-specific* modeling, see [22]. It can be summarized as "intended over-parameterization with data-based simplification". We start with the set of all potentially significant explanatory variables and then we eliminate the insignificant ones with the help of a combination of various statistical tests. For this purpose we adopt three well-known tests; Student t-test, F – ratio test for nested models and Lagrange Multiplayer test for nested models. What is important, the asymptotic distribution of the latter does not depend on the probability distribution of the model disturbance Z. It is worth emphasizing here, that the phrase "insignificant variable" has special meaning in the regression analysis. It denotes the variables which do not incorporate any valuable information into the model. Some variables which are significant from the technological standpoint may appear to be insignificant in a *regression analysis* sense. It may happen for example in the case where such variables (e.g. due to given technological procedures) change their values in very small interval, or in the situation where a given variable incorporates the same information as another variable already included in the model. For example in this study it appeared that various quantities describing the rolling process were "insignificant". Thus such potential regressors as number of rolling passes, the magnitude of rolling velocity or pressure, and various characteristics of cooling conditions are not included in presented regression models. Obviously all these process parameters are extremely important and they have great impact on the final properties of the steel plate. However in the CFRT method of rolling all this parameters are established by the implemented computer system on the base of the information provided by the employee operating the rolling mill. The information consist of the slab characteristics (first of all its chemical composition) as well as characteristics of the expected final properties of the product. The employee can also set the finishing rolling temperature. Then the rolling schedule is set up by a computer system which tracks and controls the whole rolling process. As a consequence, in the considered production scheme the rolling parameters are strongly (almost uniquely) dependent on the other variables which are included in the regression models.

To estimate the regression coefficients β_i, $i = 0,...,k$, in the final model (2) one may consider various methods suggested in literature in the context of industrial data examining, [19-22]. In this study apart from the usual Least Squares method (LS) also *least absolute deviations* method and *ridge regression* method were taken into account. However, the cross validation studies show that in the case of CFRT method of rolling the regression models based on LS-estimates give similar or even less prediction errors than the other ones. Thus, finally, the LS method was used for the regression parameters estimation. The estimated models for the parameters *Rm, Reh*, and *A* obtained in this studies are presented below.

$$\begin{gathered}\hat{Rm} = 278.31 + 839.844C + 90.129Mn + 64.8577Si + 75.8515Cr + 145.227Mo+\\ 1169.12\ Nb + 52.648Ni + 649.197V + 432.95P - 0.0219599FRT - 0.568248Th\\ \hat{Reh} = 338.999 + 384.433C + 81.245Mn + 40.6527Si + 49.6527Cu + 184.166Mo+\\ 2048.55Nb + 733.913V - 0.146888FRT - 1.14488Th\\ \hat{A} = 34.8738 - 22.2288C - 2.83348Mn - 3.99648\ Si - 36.1474Nb + 286.567Ti+\\ 0.0271583Th\end{gathered} \tag{3}$$

The symbols used in the above equations has the same meaning as in Table 1. To save article space, the standard errors of coefficient estimators are omitted in the equations (3) – their exact values will play no role in the sequel of the paper. However it is worth mentioning, that they were at least three times less than the absolute values of the appropriate estimates. The most important for the presented here approach are the *prediction errors* related to these models. The prediction error probability distribution (PED) fully describes the predictive capabilities of the regression model. In this study the PEDs were examined on the base of the PEA set containing 4000 records which, as it was described previously, were not included in the MD set. Thus the results of the study reflect the true nature of the empirical distribution of the prediction errors. The below presented Table 2 shows the statistical characteristics of the probability distributions of the errors E_{Rm} = Rm-$\hat{R}m$, E_{Reh} =Reh-$\hat{R}eh$ and E_A = A- $\hat{A}$

TABLE 2
Statistical characteristics of the empirical distributions of prediction errors E_{Rm}, E_{Reh} and E_A

Empirical Distribution Characteristic		Prediction Error		
		E_{Rm}	E_{Reh}	E_A
Mean		-0.27	-0.01	-0.03
Standard Deviation		14.61	20.79	2.24
Mean Absolute Error		10.62	16.18	1.68
Quantiles of Order p	p =0.05	-22.43	-32.31	-3.52
	p =0.1	-16.47	-24.32	-2.87
	p =0.2	-10.50	-16.71	-1.83
	p =0.3	-6.48	-10.89	-1.06
	p =0.4	-3.25	-6.24	-0.46
	p =0.5 (Median)	-0.43	-1.34	0.07
	p =0.6	2.57	3.71	0.52
	p =0.7	5.77	9.20	1.03
	p =0.8	9.92	16.60	1.58
	p =0.9	16.82	26.29	2.40
	p =0.95	22.31	36.15	3.43

The SPFP constraints connected with the typical grades and/or customers requirements are of the following form:

$$\begin{gathered}\hat{R}m(C,Mn,Si,Cr,Mo,Nb,Ni,V,P,FRT,Th) \leqslant Rm_{max}\\ \hat{R}m(C,Mn,Si,Cr,Mo,Nb,Ni,V,P,FRT,Th) \geqslant Rm_{min}\\ \hat{R}eh(C,Mn,Si,Cu,Mo,Nb,V,FRT,Th) \geqslant Reh_{min}\\ \hat{A}5(C,Mn,Si,Nb,Ti,Th) \geqslant A_{min}\end{gathered} \tag{4}$$

where $\hat{R}m(\cdot)$, $\hat{R}eh(\cdot)$, and $\hat{A}(\cdot)$, are the linear functions of indicated variables given by (3) while the Rm_{min}, Reh_{min} and A_{min} are the minimal values of the indicated parameters determined in the specifications, Rm_{max} is the upper limit for the possible values of Rm.

It results from the equations (3) that all constraint given by (4) are linear. Because the assumed form of the cost function (1) as well as the usual form of the CC constraints are also linear it seems, that the optimization problem can be modeled as LP task. One can notice however, that the values of the coefficients in the linear expressions appearing on left-hand sides of these inequalities are *uncertain.* These values were estimated, and it means that they are realizations of some random variables (the estimators) and this fact cannot be neglected. The probabilistic nature of the constraints coefficients leads to Chance Constrained Programming formulation of our problem, see e.g. [16-17].

4. Chance Constrained Programming approach

Chance Constrained Programming (CCP) also known as Probabilistic Programming, see [15-16], is a class of stochastic optimization tasks in which some of the data may be subject to significant uncertainty. Such models are appropriate when the model parameters cannot be measured/stated without error or when they evolve over time and decisions have to be made prior to observing the entire data stream. The concept of CCP was introduced in the classical work of Charnes and Cooper [15]. Now CCP belongs to the major approaches for dealing with random parameters in optimization problems. In models built for real-world problems various uncertainties enter the inequalities describing the natural constraints that should be satisfied for proper working of a system under consideration. We adopt the approach to deal with the uncertainties connected with the models for prediction of the final mechanical properties of the steel plates.

4.1. LP versus CCP approaches to the optimization problem

Let us consider a deterministic LP problem in the following form:

minimize $f(x_1, \ldots, x_n) = c_1x_1 + c_2x_2 + \ldots + c_nx_n$
subject to the following constrains:
$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n \geqslant b_i i = 1, \ldots, m$
$x_1 \geqslant 0, \ldots, x_n \geqslant 0$

where $\mathbf{x}=[x_1,x_2, \ldots, x_n]^T$ is the decision variable vector, $\mathbf{c}=[c_1,c_2, \ldots, c_n]^T$ is a vector of the objective function coefficients, $\mathbf{A}=[a_{ij}]_{mxn}$ is the matrix of technical coefficients of the system of linear inequalities, and a vector

$\mathbf{b}=[b_1,b_2,\ldots,b_m]^T$ forms a right hand side of the constraints system.

In many real world problems it turns out that some of the elements of matrix **A** and/or vectors **b**, **c** are uncertain due to their random nature. In all such cases it is difficult or even impossible to know which solution will appear to be feasible. In such circumstances, one would rather insist on decisions guaranteeing feasibility "as much as possible". This loose term refers to the fact that in some problems constraint violation can almost never be avoided because of unexpected random events. On the other hand, after a proper estimating the distribution of the random parameters, it makes sense to call decisions feasible (in a stochastic meaning) whenever they are feasible with *high probability*, i.e. only a low percentage of realizations of the random parameters leads to constraint violation under this given decision. It leads to CCP formulation of the problem, where the deterministic constraints are replaced with a probabilistic ones in the following way:

minimize $\mathrm{E}f(x_1,\ldots,x_n)=\mathrm{E}(c_1)x_1+\mathrm{E}(c_2)x_2+\ldots+\mathrm{E}(c_n)x_n$

subject to

$\Pr[a_{i1}x_1+a_{i2}x_2+\ldots+a_{in}x_n \geqslant b_i] \geqslant q_i\, i=1,\ldots,m$

$x_1 \geqslant 0,\ldots,x_n \geqslant 0$

where $\Pr[B]$ denotes a probability of an event B, while the $q_i \in [0,1], i=1,\ldots,m$, are some prescribed values of so-called *probability levels* given by the decision maker for each constraint separately. Such a version of the problem is called the Chance Constraint Programming model with *individual* constraint probability levels.

So, according to the CCP idea, the SPFP constraints in our optimization model should take the following form:

$$\begin{gathered}
Pr[Rm(C,Mn,Si,Cr,Mo,Nb,Ni,V,P,FRT,Th) \\
\leqslant Rm_{max}] \geqslant q_{Rm} \\
Pr[Rm(C,Mn,Si,Cr,Mo,Nb,Ni,V,P,FRT,Th) \\
\geqslant Rm_{min}] \geqslant q_{Rm} \\
Pr[Reh(C,Mn,Si,Cu,Mo,Nb,V,FRT,Th) \\
\geqslant Reh_{min}] \geqslant q_{Reh} \\
Pr[A(C,Mn,Nb,Si,Ti,Th) \geqslant A_{min}] \geqslant q_A
\end{gathered} \tag{5}$$

where q_{Rm}, q_{Reh} and q_A are the prescribed probability levels given by the decision maker and *Rm*, *Reh*, and *A* are the random values of the considered parameters obtained for the fixed values of the indicated variables.

Generally speaking, stochastic optimization problems belong to the most difficult problems of mathematical programming, see [17,23]. However, some of them can be transformed to an equivalent deterministic optimization task, see [16,17]. Our problem belongs to the latter class of problems. Indeed, it can be quite easily verified that the inequalities (3) hold if and only if the following inequalities are satisfied, see e.g [16]:

$$\begin{gathered}
\hat{Rm}(C,Mn,Si,Cr,Mo,Nb,Ni,V,P,FRT,Th) \\
\leqslant Rm_{max} - Q^1_{Rm} \\
\hat{Rm}(C,Mn,Si,Cr,Mo,Nb,Ni,V,P,FRT,Th) \\
\geqslant Rm_{min} - Q^2_{Rm} \\
\hat{Reh}(C,Mn,Si,Cu,Mo,Nb,V,FRT,Th) \\
\geqslant Reh_{min} - Q_{Reh} \\
\hat{A}5(C,Mn,Nb,Si,Ti,Th) \geqslant A_{min} - Q_A
\end{gathered} \tag{6}$$

where $Q^1_{Rm}, Q^2_{Rm}, Q_{Reh}$ and Q_A denote the quantiles of the probability distributions of the prediction errors E_{Rm}, E_{Reh}, and E_A, see Table 2. The order of the quantile Q^1_{Rm} equals q_{Rm}, whilst the remaining ones have orders $1\text{-}q_{Rm}$, $1\text{-}q_{Reh}$ and $1\text{-}q_A$, respectively.

Examples of optimization problems and a comparison of the solutions obtained with the help of the deterministic and probabilistic approaches are presented in the next section.

5. Examples of an optimization problem and their solutions

It results from the inequalities (3), (6) that the decision variables in our optimization models for modeling the mechanical properties of the steel plates are the following: *C, Mn, Si, Cr, Cu, Mo, Nb, Ni, V, P, Ti, FRT, Th.* The remaining, usually residual (or tramp) elements present in the alloy and other technological or final product parameters should stay within the already observed ranges resulting from the industrial practice and technologies applied in the institution. Obviously, they should also comply with additional constraints that result from the grade standards and/or customer requirements.

Now, let us consider two examples illustrating the difference between the solutions resulting from both LP and CCP approaches.

Example 1. Let the objective function (1) in the first example take the following form:

$$CF = 5Cr + 40Mo + 20Nb + 30V \tag{7}$$

Such cost function is unitless. The weights represent actual relative impact of a given factor on the total cost. As it was mentioned before, the weights may result directly from the price of given admixture and/or the cost of the process connected with the controlling of its concentration. It should be emphasized here that the weighs in (5) and in the sequel of the paper, play only an exemplary role, because the prices of the alloying elements may be a subject of great fluctuation in the world market. The examples of such situations, where prices of some mineral commodities dramatically rose whilst other drop even more than two times can be found in the statistic data describing the mineral commodities world market, see e.g. [24].

The omission of the remaining decision variables in the formula (7) may reflect the fact that they have neglectably small impact on the total cost or that their desired (or not) level is already known and it shouldn't (or cannot) be changed. Let us assume for example , that the concentration of *Cu* and *Ni* in the alloy equals 0.11%, 0.19%, respectively. In the final optimization program it should also be taken into account that all decision variables should take values within observed ranges given in Table 1.

Now, assume that our task is to produce the plate of the final thickness 85 mm and made of the weldable fine grain structural steel which is characterized as S460M grade according the European standards EN10025-4:2004. So, finally all constraints connected with our decision variables can be formulated as follows (the units are omitted):

$$\begin{gathered} C \leqslant 0.18\ Mn \leqslant 1.70, Si \leqslant 0.60, Cr \leqslant 0.30, \\ Cu = 0.11, Mo \leqslant 0.20, Nb \leqslant 0.05, Ni = 0.19, \\ V \leqslant 0.12, P \leqslant 0.035, Ti \leqslant 0.05, \\ CEV = C + Mn/6 + (Cr + Mo + V)/5 + \\ (Cu + Ni)/15 \leqslant 0.48 \\ FRT \geqslant 755, FRT \leqslant 961, Th = 85, \end{gathered} \tag{8}$$

and

$$Rm \leqslant 680, Rm \geqslant 500, Reh \geqslant 400, A \geqslant 17 \tag{9}$$

In considered case all constraints are linear, thus, if we decide to ignore the fact that the coefficients in formulae (3) are uncertain, the optimization problem can be expressed as the following LP task:

minimize CF=20 Cr + 30 Mo + 25 V+25 Nb
s.t. constraints given by (6)
and

$$\begin{gathered} \hat{R}m(C, Mn, Si, Cr, Mo, Nb, Ni, V, P, FRT, Th) \\ \leqslant 680, \\ \hat{R}m(C, Mn, Si, Cr, Mo, Nb, Ni, V, P, FRT, Th) \\ \geqslant 500, \\ \hat{R}eh(C, Mn, Si, Cu, Mo, Nb, V, FRT, Th) \\ \geqslant 400, \\ \hat{A}5(C, Mn, Nb, Si, Ti, Th) \geqslant 17 \end{gathered} \tag{10}$$

The optimal solution of this LP problem is the following (here and in the next presented solutions, the chemical elements are expressed in %) :

C =0.18, Cr=0.02, Cu=0.11, Mn=1.6, Mo=0.01, Nb=0.0206, Ni=0.19, P =0.007, Si=0.49, Ti=0.001, V =0.001, FRT=755°C, Th=85mm.

The expected values of the parameters *Rm, Reh* and *A* resulting from this solution are: 582.6 MPa, 400 MPa, and 26.23%, respectively. The minimal cost equals 0.94. The latter value cannot be directly interpreted - it can be only used for comparisons with other solutions. So, now let us compare it with the value of the optimal cost found in the CCP version of this problem. For this purpose let us assume the probability levels q_{Rm}, q_{Reh} and q_A equal to 0.95, compare (5). Consequently, from the formulae (6) and Table 2 we obtain the following form for the CCP optimization program in this case:

minimize CF=5 Cr + 40 Mo +20 Nb +30 V
s.t. constraints given by (8)
and

$$\begin{gathered} \hat{R}m(C, Mn, Si, Cr, Mo, Nb, Ni, V, P, FRT, Th) \\ \leqslant 680 - 22.31, \\ \hat{R}m(C, Mn, Si, Cr, Mo, Nb, Ni, V, P, FRT, Th) \\ \geqslant 500 + 22.43, \\ \hat{R}eh(C, Mn, Si, Cu, Mo, Nb, V, FRT, Th) \\ \geqslant 400 + 32.31, \\ \hat{A}5(C, Mn, Nb, Si, Ti, Th) \geqslant 17 + 3.52 \end{gathered}$$

The optimal solution is: C =0.18, Cr=0.02, Cu=0.11, Mn=1.6, Mo=0.01, Nb=0.0359, Ni=0.19, P =0.007, Si=0.49, Ti=0.001, V =0.001, FRT=755°C, Th=85mm. The expected values of the parameters *Rm, Reh* and *A* in this case are equal to 582.6 MPa, 431MPa and 25.7%, respectively. The minimal cost equals 1.248 and, as we see, it is a greater value than in the LP program. It can be easily explained, because the CCP program takes into account the risk connected with the uncertainty and tries to find *less risky* values of the parameters connected with mechanical properties. However, we have to cover the cost of these *safety margins*.

Example 2. To illustrate the influence of the weights on the solutions let us consider now another cost function. Assume now that although the unit costs connected with the concentration of the primary alloying elements are relatively small, their influence on solution cannot be neglected. So, just for example, let us assume that the cost function (1) now takes the form:

CF=5 Cr + 40 Mo +20 Nb +30 V + C+2 Si+3 Mn

In this case we obtain the following solutions for the LP and CCP problems.

Solution for LP problem. Optimal values of explanatory variables: C =0.18, Cr=0.02, Cu=0.11, Mn=1.286, Mo=0.01, Nb=0.04, Ni=0.19, P =0.007, Si=0.14, Ti=0.001, V =0.001, FRT=755°C, Th=85mm. Expected values of the parameters describing mechanical properties: Rm=554.3 MPa, Reh=400.0 MPa, A = 27.8%. Minimal value of the cost function: 5.64.

Solution for CCP problem. Optimal values of explanatory variables: C =0.18, Cr=0.02, Cu=0.11, Mn=1.6, Mo=0.01, Nb=0.04, Ni=0.19, P =0.007, Si=0.14, Ti=0.001, V =0.0090, FRT=755°C, Th=85mm. Expected values of the parameters describing mechanical properties: Rm=586 MPa, Reh=431MPa, A = 26.9%. Minimal value of the cost function: 5.66.

We see again, that minimal cost value is greater in the case of CCP model. It can be also noticed that the only explanatory variables which have different values in the presented optimal solutions are: *Mn, Nb, Si, V*.

6. Conclusions and final remarks

As it can be seen in the examples above the optimal solutions found in CCP framework lead to greater cost than those obtained via related LP programs. On the other hand the CCP models for the considered optimization tasks are much more realistic. It is because the conventional LP model does not take into account the uncertainty connected with the forecasting of the values of parameters characterizing the mechanical properties of the steel plate. Consequently, the applications of LP models in metallurgical practice can be very costly due to the large probability that the desired parameter levels will not be achieved. So it seems, that such models should be avoided in industrial practice. Instead, in all cases where uncertain parameters appear in the constraints, the CCP programs should be used as models of the optimization problems.

The decision variables appearing in considered optimization models differ with respect to the extent to which they may be practically removed or controlled in steelmaking industrial practice. Some of them, e.g. *Ni* or *Cu*, once presented in molten steel, can't be removed at all. One can control their concentration in the alloy by controlling their amount in furnace charge. In the optimization models the concentration of such variables could be given in the form of equality constraint, similarly as in presented examples. Concentration of some other elements, e.g. *Cr, Mo*, can be controlled rather inefficiently under the industrial melting furnace practice and the process of control itself may be very costly. Again, the control of the amount of such elements in the furnace charge is perhaps the best solution. Yet another elements, due to participation in oxidation/reduction reactions at steel making temperatures, can usually be efficiently removable or controllable. All this facts should be reflected in a proper construction of the system of constraints as well as the form of the cost function (1). The more limitations connected with the adopted technological process are taken into account, the greater is the actual role of such optimization models. Obviously the limitations should also include those related to the controllability of element concentration and the processing cost of these "controls". The presented examples also show that the proper choice of the cost function has the fundamental importance. Thus it should be stressed again that both, the constraints and the weights determining the cost, have to be defined carefully by factory experts, and that they should be each time adapted to changing market and/or modified manufacturing technologies.

The optimization problems investigated in the paper were treated as versions of the LP task. It was possible, because in presented examples all considered restrictions appear to be linear as well as the assumed form of the cost function. As it was pointed out, such a form of restrictions is very natural in considered class of optimization problems. From computational point of view such a formulation of the problem is very convenient because the optimal solution can be found effectively via simplex method. But even if some of the constraints were nonlinear or if the utilities connected with the production cost were better reflected by nonlinear criterion function, the CCP approach could also be adopted. However in such cases usually much more sophisticated numerical techniques should be used to find the solution, see e.g. [18,23,25].

REFERENCES

[1] G.S. Dulikravich, I.N. Egorov, Robust optimization of concentrations of alloying elements in steel for maximum temperature, strength, time-to-rupture and minimum cost and weight, Conference on Computational Methods for Coupled Problems in Science and Engineering – Coupled Problems 2005, Greece, 2005. available version: http://www.iosotech.com/files/alication/Alloys-properties-optimization.pdf

[2] J. Kusiak, A. Żmudzki, A. Danielewska-Tulecka, Optimization of material processing using a hybrid technique based on artificial neural networks, Archives of Metallurgy and Materials **3**, 50, 609-620 (2005).

[3] S. Natschlager, S. Dimitrov, K. Stohl, EAF process optimization: theory and real results, Archives of Metallurgy and Materials **2**, 53, 373-378 (2008).

[4] P.K. Ray, R.I. Ganguly, A.K. Panda, Optimization of mechanical properties of an HSLA-100 steel through control of heat treatment variables, Materials Science and Engineering **A346**, 122-131 (2003).

[5] A.P. Paiva, E.J. Paiva, J.R. Ferreira, P.P. Balestrassi, S.C. Costa, A multivariate mean

square error optimization of AISI 52100 hardened steel turning, International Journal of Advanced Manufacturing Technology **43**, 631-643 (2009).

[6] T.N. S i d o r i n a, I.V. K a b a n o v, Optimization of carburizing steels for drilling tools within grade chemical composition, Metal Science and Heat Treatment **49**, 9-10 (2007).

[7] L i u X i a n g H u a, L a n H u i F a n g, D u L i n X i u, L i u W e i J i e, High performance low cost steels with ultrafine grained and multi-phased microstructure, Science China Technological Sciences **52**, 8, 2245-2254 (2009).

[8] J. T a l a r, Data mining methods – application in metallurgy, Archives of Metallurgy and Materials **2**, 52, 239-250 (2007).

[9] J. M o r a v k a, K. M i c h a l e k, B. C h m i e l, Statistical analysis of heats with targeted overheating realized in the EAF at Trinec Steelworks, Archives of Metallurgy and Materials **2**, 53, 1-8 (2008).

[10] H.B. X i e, Z.Y. J i a n g, X.H. L i u, G.D. W a n g, A.K. T i e u, Prediction of coiling temperature on run-out table of hot strip mill using data mining, Journal of Materials Processing Technology **177**, 121-125 (2006).

[11] P. P a l a n i s a m y, I. R a j e n d r a n, S. S h a n m u g a s u n d a r a m, Prediction of tool wear using regression and ANN models in end-milling operation, International Journal of Advanced Manufacturing Technology **37**, 29-41 (2008).

[12] V.K. P o t e m k i n, O.S. K h l y b o v, V.A. P e s h k o v, Complex mathematical model for predicting mechanical properties and structure of steel sheets, Metal Science and Heat Treatment **42**, 11-12 (2000).

[13] P.C.M. R o d r i g u e s, E.V. P e r e l o m a, D.B. S a n t o s, Mechanical properties of an HSLA bainitic steel subjected to controlled rolling with accelerated cooling, Materials Science and Engineering **A283**, 136-143 (2000).

[14] A.Z. G r z y b o w s k i, Z. U r b a n o w i c z, Statystyczne modelowanie własności mechanicznych blach grubych przy regulowanym walcowaniu, Materiały Konferencji Zastosowania Komputerów w Zakładach Przetwórstwa Metali, Bukowina Tatrzańska, 25-33 (1998).

[15] A. C h a r n e s, W.W. C o o p e r, Chance-constrained programming, Management Sciences **6**, 73-80 (1959).

[16] N.V. S a h i n i d i s, Optimization under uncertainty: state-of-the-art and opportunities, Computers and Chemical Engineering **28**, 971-983 (2004).

[17] A. R u s z c z y n s k i, A. S h a p i r o, Stochastic programming. Handbooks in operations research and management science, Elsevier, Amsterdam 2003.

[18] D.A. G u r y a n o v, B.N. Z a m o t a e v, I.V. R u b e z h a n s k a y a, Influence of the rolling parameters in high-temperature thermomechanical treatment on the structure and mechanical properties of steel, Steel in Translation **37**, 9, 730-732 (2007).

[19] D.A. B e l s l e y, Conditioning diagnostics: collinearity and weak data in regression, J. Wiley & Sons, New York 1991.

[20] A.Z. G r z y b o w s k i, Z. U r b a n o w i c z, Alternative methods of regression in modeling properties of steel plate – a comparative studies, [in:] Proceedings of 3^d International Conference on Parallel Processing & Alied Mathematics, Kazimierz Dolny, 533-542 (1999).

[21] I.C. D i m a, A.Z. G r z y b o w s k i, J.K. G r a b a r a, G.R. G o l d b a c h, O.R. P o p e s c u, Utilizing of mathematics-statistics methods concerning mechanic properties of heavy steel plates – dealing with the ill conditioned data – [in:] Proceedings of International Conference on Mathematical Models for Engineering Science (MMES '10) (ed. V. Mladenov, K. Psarris, N. Mastorakis, A. Caballero, G. Vachtsevanos), WSEAS Press, 277-284 (2010).

[22] E.W. F r e e s, Data Analysis Using Regression Models, Prentice Hall, New Jersey 1996.

[23] A.Z. G r z y b o w s k i, Monte Carlo optimization procedure for chance constrained programming – simulation study results, Scientific Research of the Institute of Mathematics and Computer Science **1**, 8, 39-46 (2009).

[24] T.D. K e l l y, G.R. M a t o s, Historical Statistics for Mineral and Material Commodities in the United States, Open-File Report 01-006, U.S. Geological Survey Data Series 140, (2010) available on-line: http://minerals.usgs.gov/ds/2005/140/ (July 30, 2011).

[25] P. K a l l, J. M a y e r, Stochastic linear programming – models, theory, and computation, Springer, 2011.

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