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## DYNAMIC BEHAVIOUR OF A VIBRATING JAW CRUSHER FOR DISINTEGRATION OF HARD MATERIALS

## DYNAMIKA WIBRACYJNEJ KRUSZARKI SZCZĘKOWEJ DO ROZDRABNIANIA MATERIAŁÓW TWARDYCH

One of the trends in design solutions of crushers ensuring the crushing ratio of about 30 involves the application of the vibratory-impulse action to the material to be crushed. Crushers utilising these effects are referred to as vibratory crushers. During the vibratory crushing the material to be disintegrated is subjected to the action of fast changing shearing forces, which leads to the material being crushed either by applied impulses or by fatigue action, unlike conventional crushers where the structure of the material is damaged by the applied pressure.

A dynamic analysis of the vibrating jaw crusher operation is provided and its potential application to crushing hard materials, such as alloy materials containing iron and slag from metallurgical processes are explored.

*Keywords*: crusher, dynamic, disintegration

Jednym z kierunków poszukiwań rozwiązań konstrukcji kruszarek – w których uzyskuje się stopień rozdrobnienia około 30 – jest wykorzystanie efektu udarowo – wibracyjnego oddziaływania na kruszony materiał. Kruszarki wykorzystujące tę zasadę działania nazywa się kruszarkami wibracyjnymi. Zasada wibracyjnego rozdrabniania polega na poddaniu rozdrabnianego materiału działania szybko zmiennych sił ściskających, co w rezultacie powoduje rozdrobnienie materiałów udarem, względzie na zasadzie zmęczeniowej, a nie jak w klasycznych kruszarkach, niszczenie struktury materiału zgniotem.

W opracowaniu – po przedstawieniu analizy dynamicznej pracy kruszarki wibracyjnej – wskazano na możliwości jej wykorzystania do rozdrabniania materiałów trwałych, w tym żelastostopów oraz żużli metalurgicznych.

## 1. Introduction

Jaw crushers are widely used crushing machines. They are used for disintegration of hard and medium-hard materials to obtain various product size. They are robust, their design and construction is relatively simple, they are easy to operate and maintain. Alongside those obvious advantages, they exhibit a number of drawbacks, too, including: low crushing ratio (of the order of 5), large inertia loading to foundations, large mass.

One of the trends in design solutions of crushers ensuring the crushing ratio of about 30 involves the application of the vibratory – impulse action to the material to be crushed. Crushers utilising these effects are referred to as vibrating crushers. During the vibratory crushing the material to be disintegrated is subjected to the action of fast changing shearing forces, which leads to the material being crushed either by applied impulses or by fatigue action, unlike conventional crushers where the structure of the material is damaged by the applied pressure.

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## 2. Dynamic analysis of a jaw in a vibrating jaw crusher

The analysis of design solutions used in vibrating jaw crushers reveals that in most cases jaw vibrations are induced by two- or four-mass inertia vibrators [2], [3]. Vibrating jaw crushers operate mostly below the range of natural frequencies $\omega_0/\omega = 0.6$-$0.8$ ($\omega_0$ – natural frequency of vibrations) so the jaw lead increases with an increase in the jaw vibration frequency, in accordance with the first branch of the resonance frequency plot [4] for induced vibrations of a single DOF (degree-of-freedom) system.

The jaw in a vibratory jaw crusher (Fig. 1) complete with springs constitutes a vibrating system, which can be approximated by a model of a single DOF system, shown in Fig. 2. The requirement stipulating high frequencies of jaw crushers' vibrations encourages the use of steel springs, with a very low damping ratio. Therefore, the damping in the considered vibrating system can be neglected. The equation of motion of a jaw in a vibrating jaw crusher shown schematically in Fig 2 can be written as [2]:

$$J_0 \overset{00}{\varphi} + k_z \cdot l^2 \sin \varphi = b \cdot P_0 \sin \omega t \qquad (1)$$

where:

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$J_0 \left[ kg \cdot m^2 \right]$ – total inertia moment of the jaw with respect to the axis of rotation,

$\overset{00}{\varphi} \left[ s^{-1} \right]$ – angular acceleration,

$k \left[ Nm^{-1} \right]$ – equivalent elasticity factor of the battery of compressions,

$\varphi$ [rad] – angle of jaw's rotation,

$l$ [m] – distance between the spring attachment points and the jaw's rotation axis,

$P_0$ [N] – amplitude of the excitation force,

$\omega$ [$s^{-1}$] – frequency of the applied excitation force,

$t$ [s] – time,

$b$ [m] – distance between the point of force application and the rotation axis.

Since the angle of the jaw's rotation $\varphi$ in this type of crusher machines does not exceed 0.085 rad (5°), it can be assumed that:
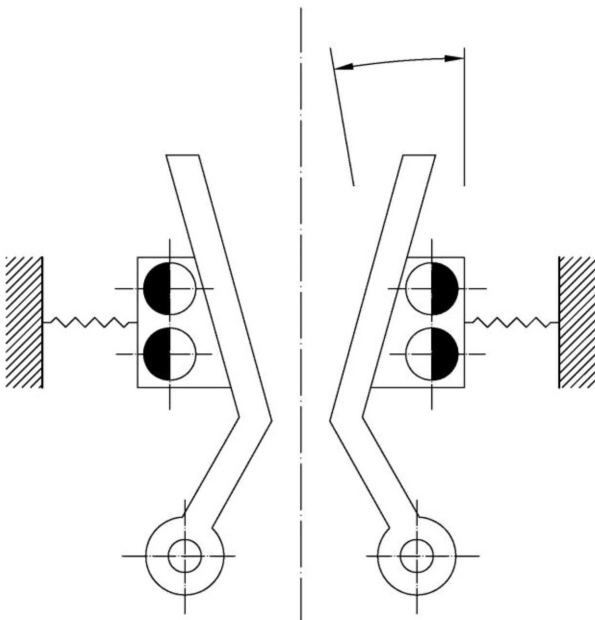
$$\sin \varphi \cong \varphi \tag{2}$$



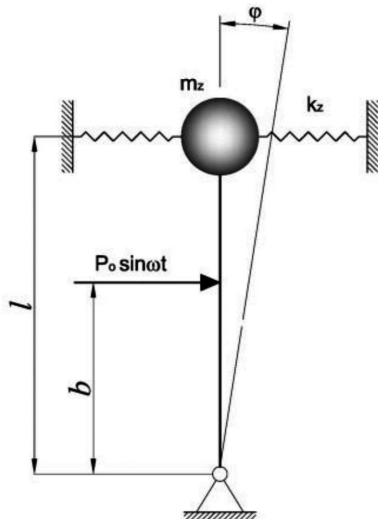Fig. 1. Schematic diagram of a vibrating jaw crusher with jaws fixed on lower joints



Fig. 2. Physical model of a jaw in a vibrating crusher

Substituting (2) into 1) yields the differential equation of the jaw's motion with respect to the rotation axis:

$$J_0 \overset{00}{\varphi} + k_z \cdot l^2 \cdot \varphi = b \cdot P_0 \sin \omega t \tag{3}$$

For the steady-state motion, the solution to the differential equation (3) is the function:

$$\varphi = \varphi_0 \sin \omega t \tag{4}$$

Substituting (4) and into (5) and multiplying by „$l$" yields a formula expressing the vibration amplitude at the spring attachment point during the idle run:

$$A_k = \frac{aP_0}{l \left( k_z - \frac{J_0}{l^2} \omega^2 \right)} \tag{5}$$

As shown in Fig. 2, the amplitude of jaw vibration on the level of the discharge can be expressed by the formula:

$$A_e = \frac{r}{l} \cos \beta \cdot A_k \tag{6}$$

Substituting (6) into (5) yields the formula expressing the amplitude of jaw vibration on the level of the discharge opening.

$$A_e = \frac{aP_0 \cdot r}{l^2 \left( k_z - \frac{J_0}{l^2} \omega^2 \right)} \cos \beta. \tag{7}$$

Substituting the following expressions:

$$P_0 = m_0 r_0 \omega^2 \tag{8}$$

$$\omega = 2\pi n \tag{9}$$

where:

$m_0 r_0 \left[ kg \cdot m \right]$ – product of undefined mass and its radius

$n [s^{-1}]$ – frequency of the jaw's vibration

into (7) yields the formula governing the vibration amplitude "$A_e$" on the level of the discharge:

$$A_e = \frac{4\pi^2 al \cdot m_0 \cdot r_0 n^2 r}{l^2 \left( k_z - \frac{J_0}{l^2} 4\pi^2 n \right)} \tag{10}$$

The constructional parameters of the crusher being known, the equation (10) can be applied to find the amplitude of the jaw's vibration on the level of the discharge depending on frequency of the jaw's vibration.

Relationships (4) and (5) give the angular velocity of the jaw whilst formula (10) has relevance to the crusher's performance.

## 3. Defining the necessary conditions to initiate the vibratory or impulse crushing process

In order to define the conditions necessary to induce the stresses capable of crushing the material it is necessary to determine the motion of the mass of material to be crushed and variations of stress in time in the cross-section $x = 0$ and $x = l$ until the instant the jaws get separated from the crushed material (Fig. 3). This motion is governed by the following equation [1] [5]:

$$\frac{\partial^2 u(x,t)}{\partial t^2} - a^2 \frac{\partial^2 (x,t)}{\partial x^2} = 0 \tag{11}$$

with the initial conditions:

$$u(x, o) = 0$$

$$\frac{\partial u(x,t)}{\partial t}\Big|_{t=0} = \begin{cases} 0 \text{ dla } x > 0 \\ V_0 \text{ dla } x = 0 \end{cases} \qquad (12)$$

and the boundary conditions:

$$m\frac{\partial^2 u(o,t)}{\partial t^2} = AE\frac{\partial u(x,t)}{\partial x}\Big|_{x=0}, \qquad (13a)$$

$$u(h, t) = 0. \qquad (13b)$$

The general solution to equation (11) is given as:

$$u(x, t) = \varphi\left(t - \frac{x}{a}\right) + \psi\left(t + \frac{x}{a}\right). \qquad (14)$$

Substituting (14) to the boundary condition (13.b), we obtain:

$$\psi\left(t + \frac{h}{a}\right) = \varphi\left(t - \frac{h}{a}\right). \qquad (15)$$

As t can assume any arbitrary value, it can be thus written:

$$\psi(z) = -\varphi\left(z - \frac{2h}{a}\right), \qquad (16)$$

where the argument z may assume any value.

Recalling (16) and (14) and replacing z by relevant arguments from (14), we get the displacement formula:

$$u(x, t) = \varphi\left(t - \frac{x}{a}\right) - \varphi\left(t + \frac{x}{a} - \frac{2h}{a}\right) \qquad (17)$$

Substituting (17) into the boundary condition (13a) and rearranging, we get:

$$\varphi''(t) + \frac{AE}{ma}\varphi'(t) = \varphi''\left(t - \frac{2h}{a}\right) - \frac{AE}{ma}\varphi'\left(t - \frac{2h}{a}\right). \qquad (18)$$

Using the relationship (18) and the initial conditions, the form of the function $\varphi$ is gradually defined and, recalling (14), the motion of particular cross-sections of the rod [1] [5] can be expressed as:

$$0 \leqslant t \leqslant \frac{2h}{a}$$

$$u(x, t) = \frac{V_0}{EA,}{}^{\left[1 - e^{-\frac{EA}{ma}V_0 t}\right]}, \qquad (19)$$

$$\sigma_{(x,t)} = E\frac{\rho u(t, x)}{\partial x} = -\rho a V_0 e^{-\frac{EA}{ma}t}. \qquad (20)$$

To emphasise the qualitative effects of the impulse action, let us briefly analyse the stresses at characteristics points.

For *t = 0* and *x = 0*, recalling (17) we get:

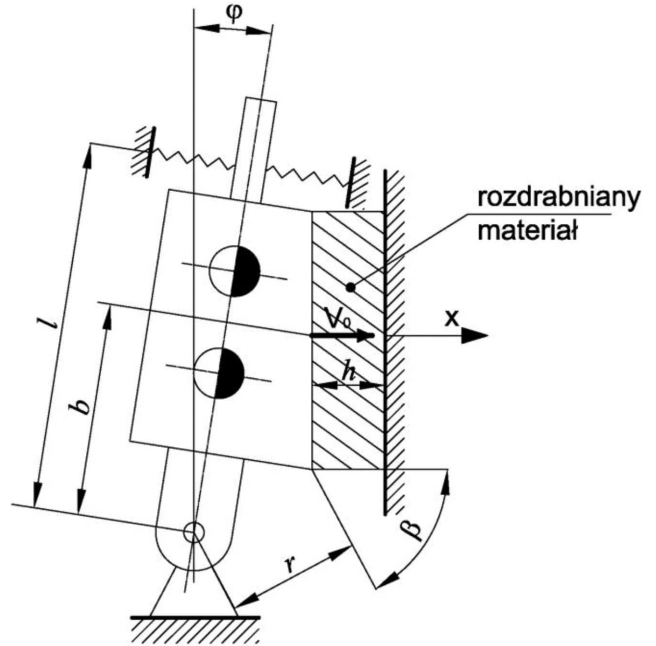$$\sigma_0 = \rho a V_0. \qquad (21)$$



Fig. 3. Geometric diagram of the jaw at the instant it touches the crushed material

It appears (see 21) that stress at the moment of applied impulse is not related to the mass of the impacting body. Of major importance is the velocity $V_0$ to which the stress is proportional. It can be concluded, therefore, that there exists some critical impulse velocity $V_k$ at which the yield point of the material gets exceeded. For example, let us calculate $V_k$ for a specified material:

$$a = \sqrt{\frac{E}{\rho}} \qquad (22)$$

$$V_k = \frac{\sigma_H}{\rho \cdot a} = \frac{\sigma_H}{\sqrt{E\rho}} \qquad (23)$$

for $t = \frac{h}{a}$, $x = h$ (the point where the stress wave 'meets' the opposite end) is obtained from formula (20b) [1], yielding:

$$\sigma = -2\rho a V_0$$

The stress experienced by the material next to investigated walls increased two-fold in relation to the initial level.

To achieve the required crushing effect, the geometric and operational parameters of the jaw crushers should be controlled such that at the instant the jaw hits the material, its peripheral velocity should become $V_0 = V_K$.

## 4. Crushing capacity of vibrating jaw crushers

To confirm the adequacy of the applied crushing method, the efficiency of the vibrating jaw crusher was tested in the laboratory set-up (Fig. 4).
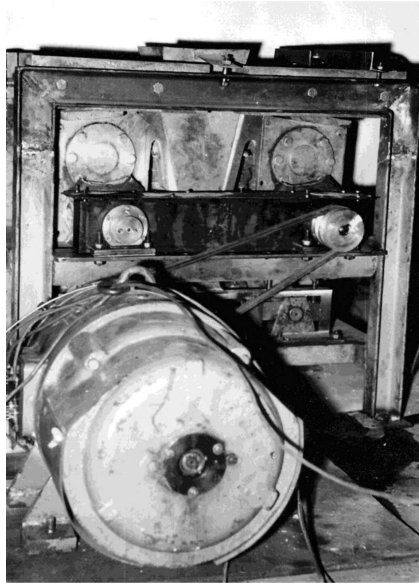
Fig. 4. Vibrating jaw crusher

This study is limited in scope and hence the presented results are restricted to the crusher's capacity in the function of jaw vibration frequency during the crushing of chamotte bricks with the grain size 40÷50 mm in relation to the dimension of the discharge (Fig. 5).
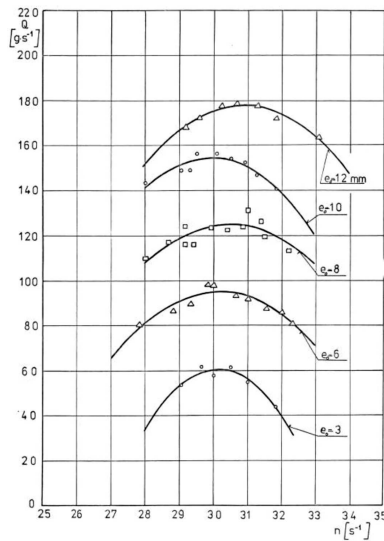


Fig. 5. Crusher capacity in the function of jaw vibration frequency for chamotte brick with the grain size 40÷50 mm

The dependence of a jaw crusher capacity in the function of jaw vibration frequency is shown in Fig. 6, for two grades of sulphur ores (grade 40÷30 – plot 1), (grade 50÷40 – plot 2) and for the fixed size of the discharge.

In the case of chamotte bricks, the maximal crushing capacity for the investigated discharge size „$e_0$" coincides with the range of jaw vibration frequencies $n = 30 \div 31$ [$s^{-1}$]. Similar patterns are obtained for sulphur ores. In both cases the maximal capacities would coincide with the frequency range $n = 30, 3 \div 32 s^{-1}$. Such narrow frequency ranges indicate that

the effect of the discharge size on the optimal performance is minimal for the frequency range $n = 30 \div 31\ s^{-1}$, at which the optima are registered for the chamotte bricks, the jaw lead during the idle run would fall in the interval $S_e = 0,8 \div 0,925\ min$.
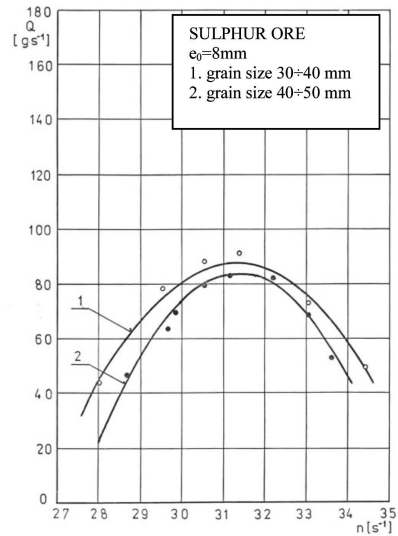


Fig. 6. Crusher capacity in the function of jaw vibration frequency for sulphur ore

## 5. Summing-up

The final effect of the research efforts is to determine the capacity of vibratory crushers for the practicable and feasible frequency ranges. For that purpose the dynamic analysis was performed of the crusher jaw behaviour and the necessary conditions were defined that would trigger the vibratory-impulse process. These considerations allowed for estimating the required technical parameters of a vibratory jaw crusher, necessary to implement the crushing process. The vibratory crusher being designed and fabricated, crushing tests were performed on various materials, including the crushing capacity tests. Results of laboratory testing of operational and constructional parameters of vibratory jaw crushers required for effective disintegration of various types of materials are provided and crushing capacity can be determined accordingly.

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