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COMPUTER SYSTEM FOR IDENTIFICATION OF MATERIAL MODELS ON THE BASIS OF PLASTOMETRIC TESTS

SYSTEM KOMPUTEROWY DO IDENTYFIKACJI MODELI MATERIAŁOWYCH NA PODSTAWIE TESTÓW PLASTOMETRYCZNYCH

The paper describes the hybrid computer system dedicated to identification of models of materials subjected to thermomechanical processing. The functionalities of the system consist of plastometric tests data processing and application of the inverse analysis. The latter functionality is realized unconventionally, instead of the finite element method the metamodel is implemented using artificial neural network. The metamodels, used for simulations of the plastometric tests, are imported to the proposed computer system as external plugins, what guarantees flexibility and possibility of further development. On the other hand, application of rich optimization libraries assures the best possible solution of the problem. Basic principles of the inverse analysis with metamodels and mentioned optimization procedures are described in the paper. Selected examples of identification of models for various metallic materials recapitulate the paper.
Keywords: inverse analysis, metamodel, computer system, plastometric test

Artykuł przedstawia system komputerowy dedykowany identyfikacji modeli materiałów poddanych obróbce cieplnomechanicznej. Funkcjonalność systemu obejmuje przetwarzanie danych pomiarowych oraz analizę odwrotną. W przeciwieństwie do rozwiązań konwencjonalnych opartych o metodę elementów skończonych, podstawę rozwiązania stanowi metamodel do symulacji prób doświadczalnych wykonywanych dla identyfikacji modeli materiałów. Metamodele zrealizowane z użyciem sztucznych sieci neuronowych są importowane do systemu jako tzw. wtyczki, co zapewnia elastyczność i możliwość rozwoju rozwiązania. Ponadto, wykorzystanie bogatych bibliotek metod optymalizacji zapewnia uzyskanie najlepszego z możliwych rozwiązań. Podstawowe zagadnienia analizy odwrotnej z użyciem metamodeli oraz wspomnianych metod optymalizacji opisane zostały w niniejszym artykule. Przedstawiona została również architektura stworzonego systemu komputerowego oraz przykłady jego wykorzystania do identyfikacji parametrów modeli wybranych materiałów metalicznych.

1. Introduction

Increasing needs for models, which enable simulation of complex phenomena occurring in materials during processing, was the motivation for this work. Maintaining reasonable computing times was the main limitation in development of these models. Models of various complexity of mathematical formulation and of various predictive capabilities are available now. Accuracy of simulations is determined by proper selection of the model to a particular task and by proper identification of the model. The latter aspect is the subject of the present work. Inverse analysis combined with plastometric tests is commonly used for identification of rheological models of materials [1]. Inverse approach connects simulation (using usually finite element (FE) method) with experiments and optimization techniques [2]. Thus, this approach requires very long computing times. Beyond high costs of computations, inverse method has to cope with several other problems [3], see Table 1. Methods of dealing with these problems are given in the second column of this table.

TABLE 1
Problems with the inverse analysis and methods to solve these problems

Problems	Methods to solve the problems
long computing times, lack of uniqueness, lack of convergence, local minima, ill-posed problem	metamodelling, sensitivity analysis, distributed computing modelling reduction techniques regularization, statistical representation, Gaussian process

Solving of particular problems in Table 1 is in the field of research of scientists in various laboratories in the world. Improvement of the efficiency of calculations and decreasing the computing time is the general objective of this paper. Inverse algorithm developed in [1] and validated in [4,5] was used as a basis for the solution in this work. Theory of application of the inverse method with metamodelling is described in earlier papers [6,7]. The hybrid computer system, which allows

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efficient application of the inverse analysis for inexperienced users is the particular objective of the present paper.

2. Model of plastometric test

2.1. Plastometric test

Axisymmetrical compression test was selected to estimate the flow stress of the materials investigated in the present work. The tests were performed on the Gleeble 3800 simulator in the IMZ Gliwice. During the experiments loads in function of tool displacements were measured. Three steels with chemical composition given in Table 2 were selected for testing of the system. IF steel has an ultra low carbon level (typically less than 50 parts per million), and the most of the interstices of a primarily iron matrix in this steel are not occupied. As such, it is called interstitial-free (IF) steel. These primarily ferritic (iron) is very formable and is used in automotive industry. TRIP steel is a high-strength steel typically used in the automotive industry [8], as well TRIP stands for TRansformation Induced Plasticity. TRIP steel has a triple phase microstructure consisting of ferrite, bainite, and retained austenite. During plastic deformation and straining, the metastable austenite phase is transformed into martensite. This transformation allows for enhanced strength and ductility. Plastometric tests performed for the IF and TRIP steels are described in [9] and are not repeated here. The third investigated steel was grade 16NiCrMo13, which is used mainly for cold forged products. During manufacturing it is a subject to hot rolling and controlled cooling in STELMOR system and is further processed by cold forming. Plastometric tests for the 16NiCrMo13 steel were performed in the present project and they were composed of compression of axisymmetrical samples measuring $\varphi 10\times 12$ mm at temperatures 850, 900, 100, 1100 and 1150°C, and strain rates 0.1, 1, 20, 100 and 200 s^{-1}.

TABLE 2

Chemical composition of investigated steels, wt%

Steel	C	Si	Mn	S	P	Ni	Cr	Mo	Cu
IF	0.003	0.023	0.15	0.012	0.017	-	-	-	-
TRIP	0.2	1.52	0.25	0.0014	0.001	1.5	-	-	-
16NiCrMo13	0.142	0.29	0.50	<0.002	0.008	3.10	0.98	0.28	0.04

Model of the plastometric tests is needed as a direct problem model in the inverse analysis [1]. Two models of the test are presented below.

2.2. Thermal-mechanical finite element model

Finite element (FE) method is commonly used as direct problem model in the inverse analysis of plastometric tests. In the present work the FE method is substituted by the metamodel based on the Artificial Neural Network. However, since this FE model was used for training the metamodel, a brief description of the FE solution is given below. This is well known thermal mechanical flow formulation, which is described in numerous publications [10-12]. Code described in [12] was used in the present work. The approach follows the extreme principle, which states that for a plastically deforming body of volume Ω, under tractions τ prescribed on the part of the surface Γ_t and velocity v prescribed on the remaining parts of surface Γ_v, under constraint $\dot{\varepsilon}_V = 0$, the actual solution minimizes the functional:

$$J = \int_{\Omega} (\sigma_i \dot{\varepsilon}_i + \lambda \dot{\varepsilon}_V)\, d\Omega - \int_{\Gamma_t} \tau^T \mathbf{v} d\Gamma_t \tag{1}$$

where: λ – Lagrange multiplier, σ_i – effective stress which, according to the Huber-Mises yield criterion, is equal to flow stress σ_p, $\dot{\varepsilon}_i$ – effective strain rate, $\dot{\varepsilon}_V$ – volumetric strain rate, $\tau = \{\tau_x, \tau_y\}^T$ – vector of boundary traction, $\mathbf{v} = \{v_x, v_y\}^T$ – vector of velocities,v_x, v_y – components of velocity vector, τ_x, τ_y – components of external stress, which in metal forming processes is a friction stress.

In the flow formulation, the vector of strain rates $\dot{\boldsymbol{\varepsilon}}$ is related to the vector of stresses σ by the Levy-Mises flow rule:

$$\boldsymbol{\sigma} = \frac{\sigma_p}{3\dot{\varepsilon}_i} \dot{\boldsymbol{\varepsilon}} \tag{2}$$

The mechanical part is coupled with the FE solution of thermal problem [12]. The thermal-mechanical problem is solved in a typical finite element manner. Discretization of equation (1) and differentiation with respect to the nodal velocities and to the Lagrange multiplier yields a set of non-linear equations, which is solved by the Newton-Raphson linearization method. Flow stress σ_p is the only one mechanical material parameter in this method of simulation. In conventional approaches to modelling of metal forming processes the flow stress is introduced as a function of the temperature, strain and strain rate. In the developed system three equations were selected to describe this function. Two simplifications of the Hansel-Spittel equation [13]:

$$\sigma = A\varepsilon^n \exp(-q\varepsilon)\, \dot{\varepsilon}^m \exp(-\beta T) \tag{3}$$

$$\sigma = A\varepsilon^n \exp\left(\frac{q}{\varepsilon}\right) \dot{\varepsilon}^m \exp(-\beta T) \tag{4}$$

and model proposed in [13]:

$$\begin{aligned} \sigma = \sqrt{3}\Bigg[& a\varepsilon^n \exp\left(\frac{\beta}{T}\right) \exp(-q\varepsilon) + \\ & [1 - \exp(-q\varepsilon)]\, a_{sat} \exp\left(\frac{\beta_{sat}}{RT}\right)\Bigg] \left(\sqrt{3}\dot{\varepsilon}\right)^m \end{aligned} \tag{5}$$

where: T– temperature in °C, ε – effective strain, $\dot{\varepsilon}$ – effective strain rate.

All the equations (3), (4) and (5) contain coefficients, which characterize the material. These coefficients are estimated on the basis of the experimental results and calculations described in the next section. The three equations are implemented in the developed system but the results for equation (3) only are presented below.

3. Inverse with metamodel

Inverse solution with the metamodel, which is the basis of the computer system developed in the present work, is described in this chapter.

3.1. Metamodel

The FE models are time consuming, in particular in connection with optimization methods in the inverse analysis. Lower computing costs are possible to achieve by applying the metamodel instead of the FE model. Metamodel is an abstraction created on the basis of lower level model of the analysed process built with an application of the selected methods of mathematical modelling [14]. Any approximation of the model of the real process that gives reasonably reliable, approximate description of the considered process and allows significant decrease of the computing costs, can be considered as a metamodel. Various techniques can be used to build a metamodel, and the artificial intelligence techniques, in particular Artificial Neural Networks (ANN), are the most frequently used. The ANN is capable to reproduce very complex relations, while the computing costs remain low.

Multi Layer Perceptron (MLP), which is frequently used in modelling of static processes, was applied in the present work as the metamodel of the axisymmetrical compression test. The metamodel consists of the set of artificial neural networks. Each network predicts the load value recorded for one, specific tool displacement. The model was tested for various numbers of intervals and division of the total displacement into 10 intervals was selected in the developed system.

The number of neurons in the input layer corresponds to the number of the parameters of the flow stress equation and parameters of the test conditions (temperature, strain rate, friction coefficient), while a single neuron of the output layer corresponds to the load value of the compression test. The neural network is taught outside of the system and can be performed in different external computer application. Therefore, the number of hidden layers and the number of neurons in the hidden layers are selected according to used tools, their teaching and normalization methods. A root mean square error (RMS) is usually applied as a measure of the accuracy of the neural network metamodel. Finally, the configuration of the network has to be exported to an output text file, which includes number of layers and neurons, biases, weights, normalization ranges, learning ranges and activation functions. Such text file is imported by software being integral part of the proposed system, allowing to create functional module, which simulates behaviour of specified neural network in form of system's plugin. Implementation and usage of the plugins is described in details in section 4.

Trained metamodel predicts the loads during compression test based on the information regarding the characteristic material parameters (parameters of the flow stress equation) and the conditions of the test (friction coefficient, temperature and strain rate). The metamodel of the axisymmetrical compression test was built. The metamodel consists of ten different ANN. Eight parameters were selected as an input signals for each network: friction coefficient, temperature, effective strain rate and five coefficients in Hansel-Spittel equation (3). The compression force F for the specified tool displacement was the ANN output signal. The ANN was designed with typical MLP neural network and trained using the data sets of 5080 records (one record consists of eight input signals and one output signal) and 1270 records was used to test the model. As mentioned earlier a root mean square error (RMS) was used as a measure of the accuracy of the neural network model. Several tests were performed to adjust optimal topologies of the networks used in metamodel. The errors were calculated as:

$$e_1 = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{F_i^{ANN} - F_i^E}{F_{i\max}^E - F_{i\max}^E}\right)^2} \tag{6}$$

$$e_2 = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(F_i^{ANN} - F_i^E\right)^2} \tag{7}$$

where: n– number of tests, F_i^{ANN}, F_i^E – force calculated by the ANN and measured, respectively, $F_{i\max}^E$, $F_{i\min}^E$ – maximum and minimum force in the experiment, respectively.

The topologies and activation functions of neural networks as well as errors are presented in Table 3. The errors, both absolute and relative values, are relatively low and metamodel can be used in inverse analysis instead of the FE model. Three activation functions, which have been used in topologies of neural networks, are shown in Figure 1.

TABLE 3
The topologies of neural networks, activation functions and errors

Force	Structure	Activation functions	e1[%]	e2
F1	8-16-5-1	logsig-tansig-tansig-purelin	0,1661	0,4735
F2	8-18-8-1	tansig-logsig-tansig-purelin	0,161	0,4754
F3	8-14-4-1	tansig-tansig-tansig-purelin	0,3215	0,9437
F4	8-26-8-1	tansig-logsig-tansig-purelin	0,4057	1,1713
F5	8-23-3-1	tansig-tansig-tansig-purelin	0,25	0,7106
F6	8-15-2-1	tansig-logsig-logsig-purelin	0,2921	0,8211
F7	8-30-10-1	tansig-tansig-tansig-purelin	0,2426	0,676
F8	8-25-10-1	logsig-tansig-logsig-purelin	0,3125	0,8769
F9	8-16-1-1	logsig-tansig-tansig-purelin	0,6942	2,0167
F10	8-27-7-1	tansig-logsig-logsig-purelin	0,6564	2,1116

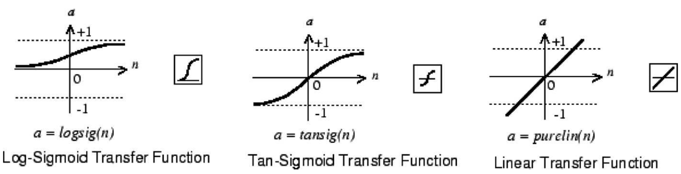

Fig. 1. Activation functions, which have been used in topologies of neural networks

3.2. Substitution FE method by metamodel in the inverse approach

The identification of the axisymmetrical compression test results was performed by the inverse calculations. The problem of the parameter identification is defined as the inverse problem. The commonly used method to solve such a task, where the direct problem is formulated with a partial differential equations, is to transform it to the optimization task with the goal function defined as the distance between calculated and measured data.

Details of the conventional inverse algorithm used in the present work are given in [1]. Simulations of the plastometric tests performed with the FE model described in section 2.2 give accurate results concerning inhomogeneities of strain/stress and temperature fields. The effect of these inhomogeneities, which are due to influence of friction in the die–specimen contact surface, as well as to the heat generation caused by friction and deformation work, cane be eliminated when the FE model is used. On the other hand, combination of the FE computations with optimization techniques is extremely time consuming. The inverse analysis, requiring the iterative recalculations of the FE models, makes the whole identification procedure useless from the practical point of view. Lowering high computing costs is possible to achieve by applying the metamodel defined in section 2.3, instead of the FE model. The idea of this solution is presented in [7] and it is shown briefly in Figure 2. Substitution of the FE model by the metamodel is demonstrated by the solid line part of the flow chart in this figure. Artificial Neural Network (ANN) described in section 3.1 was used as the metamodel in the present work. In Figure 2 $F_{ij}^{c}(\mathbf{x})$ is the force calculated either by the FEM model or by the ANN and F_{ij}^{E} is the measured force.

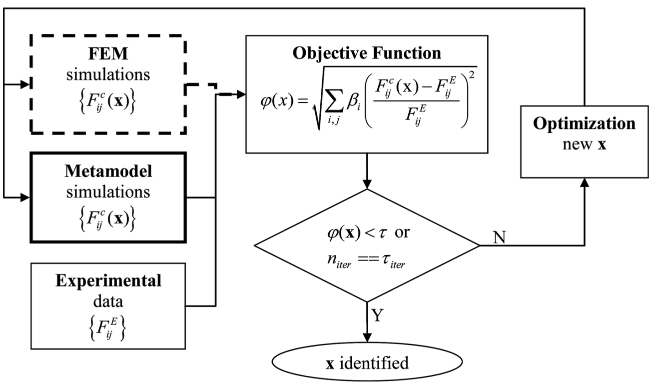

Fig. 2. Flow chart of the inverse solution with the FE model (dotted lines) and with the metamodel (solid lines)

The inverse algorithm dedicated to such identification (Figure 2) consists of the following steps:

- Performing plastometric tests and collecting the measured data – $\left\{F_{ij}^{m}\right\}$.
- Performing simulations of the experiments at constant physical conditions e.g. temperatures, strain rates, friction – $\left\{F_{ij}^{c}(\mathrm{x})\right\}$. The simulations are performed by application of conventional FEM or neural network metamodel.
- Calculation of the objective function – $\varphi(\mathbf{x}) = \sqrt{\sum_{i,j}\beta_i\left(\frac{F_{ij}^{c}(\mathrm{X})-F_{ij}^{m}}{F_{ij}^{m}}\right)^2}$. The function is defined as the mean square root error between measured and calculated loads, where: **x** is a vector of the parameters of the flow stress equations (3) or (4) or (5).
- Checking of the stop condition – usually two combined conditions are applied i.e. threshold condition τ (optimization runs until $\varphi(\mathrm{x}) < \tau$) or certain number of iteration τ_{iter} is obtained (optimization ends when $n_{iter} == \tau_{iter}$). The values $\tau = 0.001$ and $\tau_{iter} = 100$ were used in the present work.
- Running optimization procedure to determine new values of vector **x** (the parameters of the material flow stress model), which are further passed to the computing procedures of FEM or metamodel.

4. Description of the system

Application of the inverse software requires time consuming data processing and, what has already been mentioned, costly computations. Therefore, computer system was developed to enable fast data processing and inverse calculations. It was designed as multiuser and multi-access application on the basis of the Client-Server architecture, where server side consists only of database engine and its procedures. The database is composed of several tables (Figure 3 shows only the most important elements).

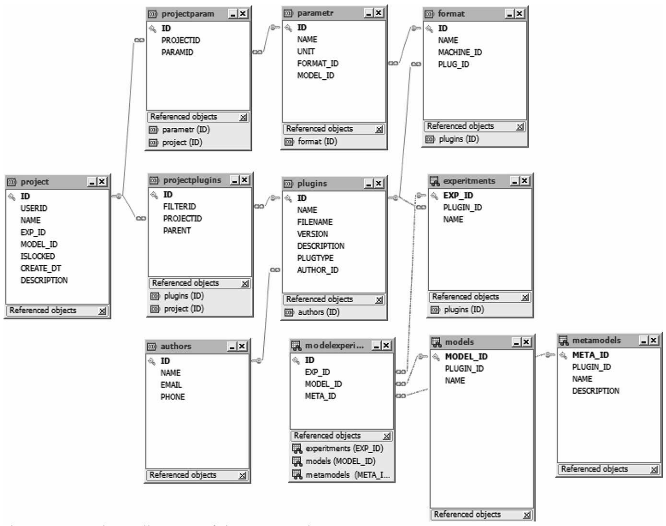

Fig. 3. Database diagram of the proposed computer system

‘Modelsexperiment’ table, which joins together ‘model’, ‘experiments’ and ‘metamodels’, plays the main role in the diagram. Such solution influences directly functionality of Graphical User Interface (GUI) allowing users to define their projects. The first step to finalize project creation is selection of the type of experimental data with measurements originated from thermo-mechanical testing devices e.g. from Gleeble 3800. This provides information to the database about user’s choice and generates the list of models (e.g. equations (3), (4) or (5)), which can be identified on the basis of imported experimental data. Due to these two choices, i.e. selection of an experiment and a model, the system is able to suggest specific metamodels, which can be used instead of numerically expensive FE method. Afterwards, user confirms his choices, what causes creation of new project records in the database. Table ‘projects’ is mainly responsible for this functionality, while two additional tables i.e. ‘projectplugins’ and ‘projectparamrs’ are used to store specific information about plugins and parameters connected to the current project. Therefore, all users selections are kept together in integrated manner.

System is designed on the basis of the reflection mechanism in the .NET platform. This platform allows dynamic extension of new functionalities to the system. In consequence, the system has flexible construction, which enables easy implementation of new modules in the form of external libraries (plugins). These programs, implemented as dynamic linked libraries (dlls) can have very wide range of applications – they may be responsible for parsing of input and output files, analysis of experimental data, filtering and, finally, metamodelling. Therefore, after creation of the new project, the experimental data is further processed by different kinds of filters responsible for sampling, cutting, smoothing and many other functionalities, which are supported by mentioned external libraries stored in the 'plugins' table (connected directly to 'experiments').

The plugins, accordingly to the current requirements, can be used from the database or implemented by a user, which allows further extension of the system functionality. The plugin in its simplest form is a single class, which implements one of the following interfaces: IFilter or IPluginReadData (Figure 4). This solution allows to add new module of reading measurement data, new filter, as well as new metamodel. All plugins are integrated in the form of complex optimization loop facilitating performance of the inverse analysis with different algorithms, different values of their parameters and different data sets.

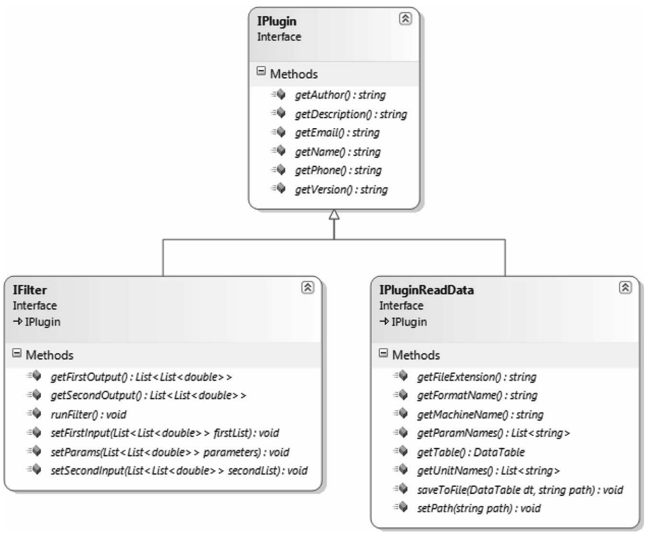

Fig. 4. Diagram of classes of the main module to read new functionalities of the system by the user

5. Results

It is shown in [7] that combination of the metamodel of plastometric tests with the inverse analysis allow for efficient identification of the flow stress models. In the present work new materials described in chapter 2 were tested and models were identified using the developed system. Selected results are presented below. Direct inverse analysis [1], which is an identification of the flow stress given in a tabular form (no equation is used), was used as a reference result. For the IF steel, the results obtained from the inverse analysis with the metamodel for equation (3) are compared in Figure 5 with the results of the direct inverse analysis and with the results of the inverse analysis performed for the model developed in the University of Sheffield [15], described also in [16]. Similar analysis was performed for the TRIP steel and the results are compared in Figure 6. The model of [15] is a complex model with 16 coefficients, therefore, identification of this model presents difficulties. It was successfully done in [7] for the IF and TRIP steels.

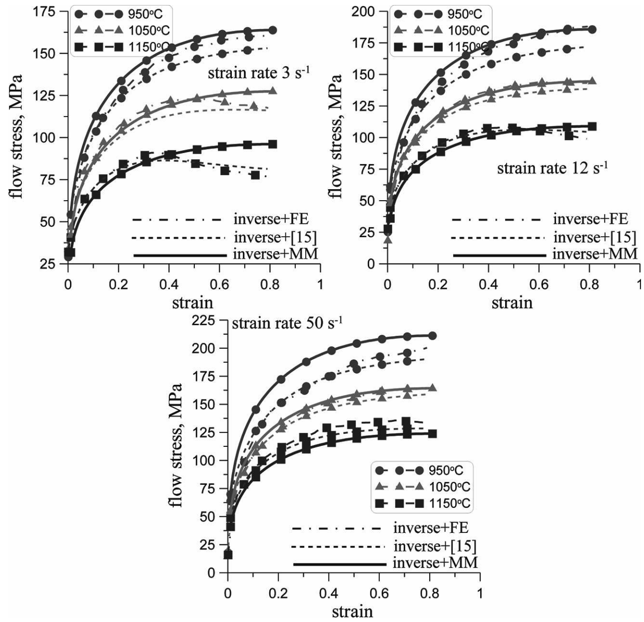

Fig. 5. IF steel – results of the comparison: direct inverse analysis with FE, inverse analysis for the model of [15] and inverse analysis with metamodel (MM) for equation (3)

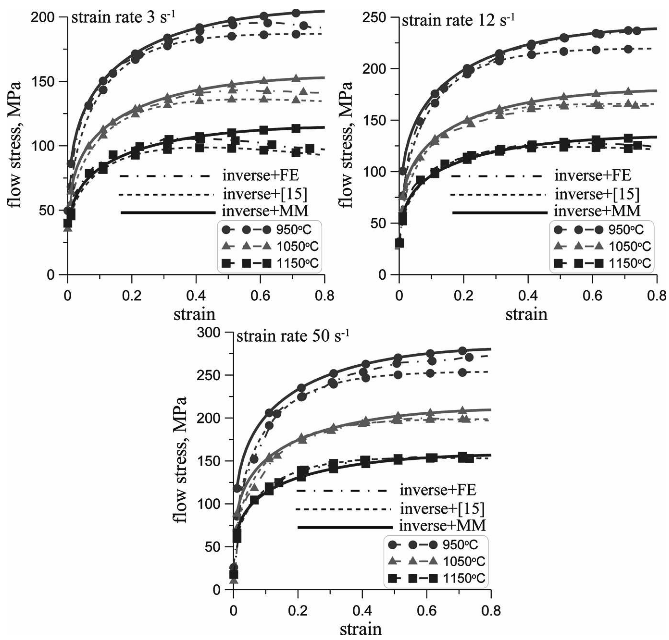

Fig. 6. TRIP steel – results of the comparison: direct inverse analysis with FE, inverse analysis for the model of [16] and inverse analysis with metamodel (MM) for equation (3)

Similar analysis was performed for the steel 16NiCrMo13. The results obtained from the inverse analysis with the metamodel for the equation (3) are compared in Figure 7 with the results of the direct inverse analysis and the results of the inverse analysis performed for the model based on the Internal Variable Method [17]. This is an advanced model, which uses numerical solution of the differential equation of evolution of dislocation populations.

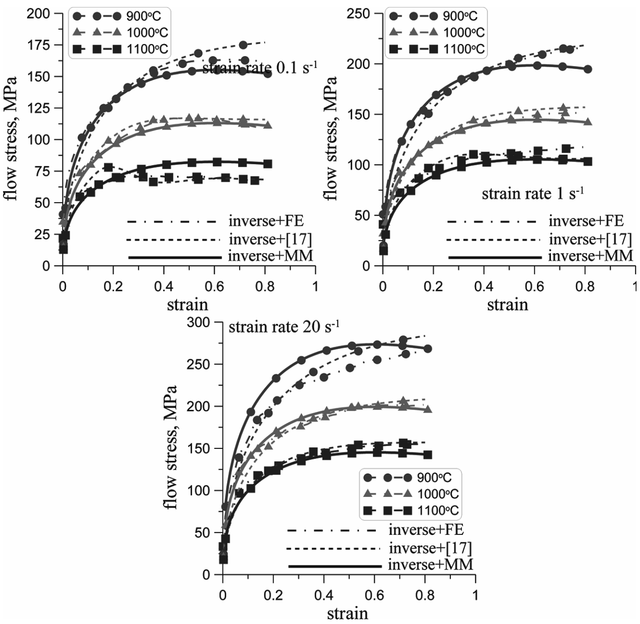

Fig. 7. Steel 16NiCrMo13 – results of the comparison: direct inverse analysis with FE, inverse analysis for the model of [17] and inverse analysis with metamodel (MM) for equation (3)

The coefficients in equation (3) obtained from the inverse analysis with the metamodel for the three investigated steels are given in Table 4. Analysis of the results in Figures 5-7 shows that very good agreement was obtained for the TRIP steel. Slightly larger discrepancies between various inverse approaches were observed for the IF and 16NiCrMo13 steels, but having in mind wide range of temperatures and strain rates the results are acceptable.

TABLE 4

The coefficients in equation (3) obtained from the inverse analysis with the metamodel for the three investigated steels

Steel	A	n	q	m	β
IF	3251.6	0.321	0.384	0.09	0.0028
TRIP	3698.2	0.242	0.254	0.112	0.00291
16NiCrMo13	7828.9	0.379	0.689	0.083	0.0035

6. Conclusions

The paper presents new approach to the inverse analysis, which, in opposite to conventional method, is based on the metamodel instead of FEM. The proposed solution allows to decrease computational cost drastically, which is caused by low computational requirements of artificial neural network in comparison to the FEM. In the case of hundreds of optimization iterations, such approach guarantees enormous yield. Simultaneously, the proposed algorithm offers similar level of quality of results as FEM-based inverse analysis, which is confirmed by results obtained for three different materials.

The metamodels, used for the purposes of inverse analysis, are created for a specific plastometric test, material model and range of model variables. Therefore, great variability of combinations occurs, what may be inconvenient to manage. Due to this reason, the computer software was designed and implemented to support management of new metamodels as well as exchange of newly added plugins between many users. The future plans assume extension of the proposed system with functionality, which allows creation and teaching of metamodel to obtain architecture of the network offering the lowest error.

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REFERENCES

[1] D. Szeliga, J. Gawąd, M. Pietrzyk, Inverse analysis for identification of rheological and friction models in metal forming, Computer Methods in Applied Mechanics and Engineering **195**, 6778-6798 (2006).

[2] M. Pietrzyk, R. Kuziak, Computer aided interpretation of results of the Jominy test, Archives of Civil and Mechanical Engineering **11**, 707-722 (2011).

[3] A. Kirsch, An introduction to the mathematical theory of inverse problems, Springer-Verlag, New York, 1996.

[4] D. Szeliga, R. Kuziak, M. Pietrzyk, Rheological model of Cu based alloys accounting for the preheating prior to deformation, Archives of Civil and Mechanical Engineering **11**, 451-467 (2011).

[5] Ł. Sztangret, D. Szeliga, J. Kusiak, M. Pietrzyk, Identification of material properties of the DP steel based on plastometric tests and on industrial hot strip rolling, Computer Methods in Materials Science **11**, 542-550 (2011).

[6] Ł. Sztangret, D. Szeliga, J. Kusiak, M. Pietrzyk, Identyfikacja modelu materiału w prawie konstytutywnym w oparciu o rozwiązanie odwrotne z metamodelem, Mechanik **84**, 32-36 (2011).

[7] Ł. Sztangret, D. Szeliga, J. Kusiak, M. Pietrzyk, Application of the inverse analysis with metamodelling for the identification of the metal flow stress, Canadian Metallurgical Quarterly **51**, 440-446 (2012).

[8] A. Grajcar, R. Kuziak, W. Zalecki, Third generation of AHSS with increased fraction of retained austenite for the automotive industry, Archives of Civil and Mechanical Engineering **12**, 334-341 (2012).

[9] E. Hadasik, R. Kuziak, R. Kawalla, M. Adamczyk, M. Pietrzyk, Rheological model for simulation of hot rolling of new generation steel strips for automotive industry, Steel Research Int. **77**, 927-933 (2006).

[10] C.H. Lee, S. Kobayashi, New solution to rigid plastic deformation problems, ASME, J. Eng. Ind. **95**, 865-873 (1973).

[11] S. Kobayashi, S.I. Oh, T. Altan, Metal forming and the finite element method, Oxford University Press, New York, Oxford, 1989.

[12] M. Pietrzyk, Finite element simulation of large plastic deformation, J. Mat. Proc. Techn. **106**, 223-229 (2000).

[13] A. Hansel, T. Spittel, Kraft- und Arbeitsbedarf Bildsomer Formgeburgs Verfahren, VEB Deutscher Verlag fur Grundstoffindustrie, Leipzig, 1979.

[14] J. K u s i a k, A. D a n i e l e w s k a - T u l e c k a, P. O p r o c h a, Optymalizacja: wybrane metody z przykładami zastosowań, PWN, Warszawa, 2009 (in Polish).

[15] S.B. D a v e n p o r t, N.J. S i l k, C.N. S p a r k s, C.M. S e l l a r s, Development of constitutive equations for the modelling of hot rolling, Materials Science and Technology **16**, 1-8 (1999).

[16] B. K o w a l s k i, C.M. S e l l a r s, M. P i e t r z y k, Development of a computer code for the interpretation of results of hot plane strain compression tests, ISIJ Int. **40**, 1230-1236 (2000).

[17] J. O r d o n, R. K u z i a k, M. P i e t r z y k, History dependent constitutive law for austenitic steels, Proc. Metal Forming 2000, eds, Pietrzyk M., Kusiak J., Majta J., Hartley P., Pillinger I., Publ. Balkema A., Krakow 2000, 747-753.

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